Mersenne and Evolving Tonal Theory
Author(s): Albion Gruber
Source: *Journal of Music Theory*, Vol. 14, No. 1 (Spring, 1970), pp. 36-67
Published by: Duke University Press on behalf of the Yale University Department of Music
Stable URL: http://www.jstor.org/stable/843036
Accessed: 11/06/2009 23:37

36

Mersenne

and

Marin Mersenne, one of the most prolific writers to have devoted his attention to music, pursued his interest in the subject during a time which was critical in the development of tonal, harmonic theory. It was in the seventeenth century – the century between Zarlino and Rameau – that the first steps were taken by theorists to formulate the basic principles which were to culminate in Rameau's epochal treatises. The present study is an examination of Mersenne's conception of harmonic theory, with a particular focus on those ideas which seem to point toward future developments in tonal theory.

Mersenne's Harmonie universelle *1 is written in a manner appropriate to its title: it is encyclopedic in scope and reveals the universal interests of its author. In reality, the work is a compilation of several treatises. There are five such divisions

Evolving Tonal Theory

ALBION GRUBER

of unequal length, each having its own pagination: the Treatise on the Nature of Sounds and Movements of Bodies (228+ pages), the Treatise on Mechanics (36 pages), the Treatise on the Voice (180+ pages), the Treatise on Consonances, Dissonances, Genera, Modes, and Composition (442+ pages), the Treatise on Instruments (559+ pages) *2, and the concluding section, New Observations Physical and Mathematical (28 pages). There are numerous occasions on which Mersenne makes cross references among these treatises; as he himself suggests in his general preface to the reader the various books which make up this monumental work may be taken in any order.

One of the striking characteristics of Mersenne's book is the great amount of space devoted to philosophical discussion, a practice also found in Zarlino's writings but one which the

French of the late seventeenth century eschew for the most part. It is interesting to note, in this connection, that Mersenne sees fit to offer an apology to his reader in the event that any moralizing might tend to bring offense. With Mersenne there is also that curious tendency to blend speculation with demonstration; one finds, for example, the value of certain musical intervals demonstrated by the alleged superiority of certain numerals, or it may be proven by a consideration of frequency ratios. Everywhere in Harmonie universelle Mersenne's universal interests—philosophy, mathematics, physics, linguistics, aesthetics, theology, as well as music—are clearly apparent.

INTERVAL GENERATION

Like many of the theorists before him, Mersenne devotes considerable space to the generation of intervals, for the most part those found in just tuning. It is worth noting that he advocates, in connection with the ratios of intervals, that the numerals represent the frequency ratio rather than the string-length ratio for each interval. In this regard Mersenne was indebted to the research of Gelileo Galilei (1564-1642). *3 Second — and because of the above — Mersenne prefers to reverse the use of the terms "arithmetic" and "harmonic" in their application to methods of string division on the monochord from their use as found in Zarlino and now considered orthodox. *4 The former term, in Zarlino's Istitutioni harmoniche for example, is defined as a division of the string in which the differences between successive ratios are equal. The latter term is defined as a division in which such differences are unequal. That is, "arithmetic" generation of the intervals results in ratios which proceed in arithmetic progression, "harmonic" generation in ratios which proceed in harmonic progression. In Zarlino's arithmetic division of the octave, for example, the string-length ratios (4:3:2) appear in arithmetic progression with the perfect fourth below the perfect fifth; in his harmonic division of the octave the string-length ratios (6:4:3) appear in "harmonic" progression with the perfect fifth below the perfect fourth.

The reason for Mersenne's reversal of these two terms of course springs from the inverse relationship which he discovered between the length of a string and its frequency: the larger the number of units measuring string-length, the smaller the number of vibrations per second. Thus, in the orthodox (Zarlino) harmonic division of the octave, the string-length ratios (6:4:3) have unequal differences proper to "harmonic" progression, but the frequency ratios that correspond to these

(2:3:4) have equal differences proper to arithmetic progression.

Once this principle is understood, various remarks of Mersenne (and later, of Rameau) can be understood correctly. Mersenne points out, for example, that the best division of the consonances is not "harmonic as has been believed, but arithmetic". Furthermore, the division of the octave with the fifth in the lower position results in the frequency ratios of 2:3:4 which, being an arithmetic progression, should be called arithmetic division of the octave — an opinion not widely shared.

THE HARMONIC SERIES

In order to appreciate properly Mersenne's conception of harmonic generation one must examine his awareness of the harmonic (or overtone) series, for his understanding of this phenomenon lies at the very foundation of his approach to consonance, octave-inversion, and the like. As Claude V. Palisca reports:

> Mersenne first noted the presence of a plurality of tones in the vibration of a single string in the early 1620's. . . . When he queried his scientific friends, as he was accustomed to do, for an explanation of the phenomenon, they put forward different theories. . . . None of these reasons satisfied Mersenne, and he acknowledged that this was the most difficult problem he had encountered in his study of sound. *5

The most frequent reference to the harmonic series made by Mersenne is that concerning the notes available to the natural trumpet. These notes, being found on the trumpet, "confirm that there are consonances in nature", and that this harmonic series contains all the consonances of music.

As for explaining why the trumpet does not proceed by means of tones and semitones, Mersenne writes:

> There is no doubt that the wind is otherwise propelled and modified in order to form the second note as it is for the first, and thus for the others; that the one which forms the second has its reflections or its vibrations twice as fast as those which produce the first, . . . which arrives by reason of the wind which is propelled with more or less violence or speed, and which consequently has its vibrations more or less frequent. *6

To be more specific as to why this increase in wind pressure results in the gapped series of the trumpet Mersenne suggests, less successfully, that "natural agents always go by the shortest path" and that "there is no addition shorter and easier than that of one to one, one to two, one to three, and so forth". Later, however, he comes closer to the truth concerning segmentation when he remarks:

> One can also give division as an explanation for these intervals, inasmuch as the octave is engendered by the division of a string into two equal parts. *7

It is in the "Treatise on Instruments", when he is concerned with stringed instruments, that Mersenne is most successful in discussing the harmonic series. In the fifth book of this work one finds proposition 11 with the title: "To determine why an open string when plucked makes several sounds at the same time." Herein Mersenne begins by mentioning that Aristotle was aware of an octave resonating above the pitch of certain sounds (Book XIX, Problem 8), but could not explain this phenomenon. He then comments upon Aristotle as follows:

> One must note that he did not know that an open string struck and sounded makes at least five different sounds at the same time, of which the first is the natural sound of the string, which serves as foundation to the others and which one only has regard for when singing and in other parts of music inasmuch as the other [sounds] are so weak that only the best ears hear them easily. *8

Mersenne further explains that demonstrating this on the monochord, which has but one string, proves that these other sounds are not coming from some other string by sympathetic vibration. As for these accompanying sounds he writes:

> Now these sounds follow the ratio of these numbers: 1, 2, 3, 4, 5 — for one hears four sounds that are different from the natural one. The first is an octave above, the second is at the twelfth, the third is at the fifteenth, and the fourth at the major seventeenth as one sees by these numbers which contain the ratios of these consonances in their smallest terms. *9

He then stresses two points:

> No sound is ever heard lower or beneath the natural sound, for they are always higher; and these sounds follow the

same progression of leaps as those of the trumpet.

Continuing his study of the harmonic series, Mersenne writes:

> Beside these four extraordinary sounds, I hear yet a fifth one higher, which I hear particularly toward the end of the natural sound and sometimes a little after the beginning. It forms a major twentieth with the natural sound, with which it is as 3:20. But I notice almost always that the twelfth and seventeenth are heard more distinctly than the others; thence it is that often one seems to hear one of these only, or that it is easily taken for the fifth or the tenth, if one is not exactly on guard. When one hears the octave and the fifteenth it is the latter which is heard more distinctly than the former.

Although the ratio 3:20 is that of the major twentieth, or compound major sixth, it is more likely that Mersenne was hearing the perfect nineteenth (1:6, a compound fifth) or, perhaps, the compound major sixth (3:10) between the third and tenth partials. He points out that the twelfth and major seventeenth are heard more distinctly than the octave duplications because they are different from the fundamental and these octave duplications. He seems aware, also, that with certain organ pipes, only the twelfth is heard above the fundamental (e. g., a stopped, cylindrical pipe produces only the odd-numbered partials). It might be helpful here to refer to the harmonic series (Example 1).

Elsewhere Mersenne realizes, at least imperfectly, that the quality of sound is due to these accompanying but faint sounds; he writes:

> . . . the sound of each string is all the more harmonious and agreeable as it causes to be heard a greater number of different sounds at the same time. *10

It is interesting to see how Mersenne wrestles with the problem of how a single, continuous string is able to produce a variety of simultaneous sounds. He comes closest to realizing the segmentation of vibrating bodies when he writes:

> Since it makes the five or six sounds of which I have spoken, it seems that it is entirely necessary that it [the string] beat the air five, four, three, and two times in the same time that it beats a single time, which is impossible to imagine. If one says that the half of the string vibrates

> twice while the entire string vibrates once, and that in the same time the third, fourth, and fifth part vibrates three, four, and five times, this is contrary to experience, which shows evidently that all the parts of the string make an equal number of vibrations at the same time since the whole string being continuous makes only a single movement, although these parts move the more slowly the nearer they are to the bridges [of the monochord]. *11

Mersenne's faith in the causal relation existing between frequency and pitch was simply not strong enough for him to accept a phenomenon which his eye could not verify! Although he seems to remain undecided as to the reason for these mysterious sounds, he gives some preference to the theory that these additional sounds are created only within the surrounding air by the string vibrating a second, third, fourth, fifth time; he writes:

> It is more probable that these different sounds arise from the different movements of the exterior air rather than those of the interior, and that these, being struck by the string, make a quantity of small movements similar to those of the water in a glass which one makes sound by running his fingers along the rim, or those of the water into which one has submerged one end of a monochord. *12

From the foregoing discussion it can be clearly seen that Mersenne was able to form a rather accurate conception of the harmonic series some sixty years before Joseph Sauveur delivered a definitive study of the principle before the French Academy of Sciences (1701). Although Mersenne failed to discover the true cause of these faint, accompanying sounds, he did realize that the complex tone of a string is somehow analogous to the series of notes available to the natural trumpet.

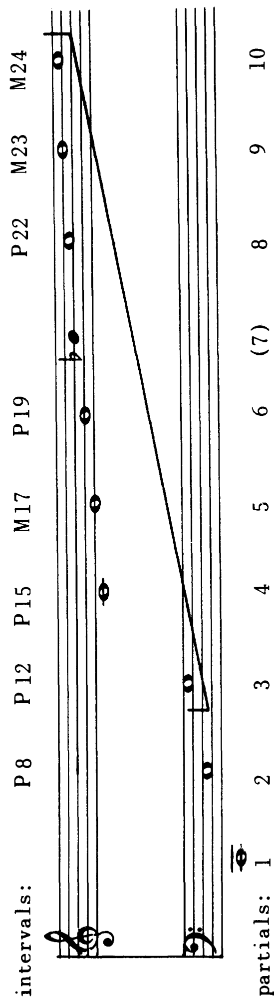
CONSONANCE AND DISSONANCE

The Consonances and Their Relative Merits

It is most characteristic of Mersenne that he begins his study of the various consonances with a rather lengthy, metaphysical discussion of the unison and its superiority to the octave. He advances many reasons, of which only some are strictly musical, for preferring the unison: the perfection of its ratio (1:1), its superiority as a final consonance, its stronger ability to cause sympathetic vibrations, and the like.

EXAMPLE

1


intervals: P8 P12 P15 M17 P19 P22 M23 M24
partials: 1 2 3 4 5 6 (7) 8 9 10

Mersenne disapprovingly recalls Zarlino's analogy in which the unison and the octave are seen as parallels of the colors white and black. Zarlino had claimed that these intervals are lacking in "color", and they are therefore not as enjoyable as the other, "less perfect" consonances. *13 This very same attitude toward harmonic "perfection" can be found in Descartes as well *14, whose Musicae compendium was well known to Mersenne. Mersenne, however, is too much of an Aristolelian to accept such a heretical attitude — one in which the very "perfection" of the unison and the octave is taken to be a fault, a blandness. He argues that if these two consonances are truly viewed as Zarlino contends, then this only demonstrates the imperfection of mankind; those who would prefer the less perfect sounds are likened to those who prefer shadowy light to the pure light of the sun. He continues by saying that, if anyone thinks that musicians who prefer diversity to unity cannot be mistaken, he must remember that these same musicians are often wrong:

> . . . as when they believe that the harmonic division of the octave [Mersenne's definition of "harmonic"!] is more agreeable than the arithmetic division; that the fifth is as good or better than the twelfth; that compositions in several parts are better than simple songs [in unison]. *15

Of such men Mersenne adds: "Impossible is it for them to quit their numerous errors, so great is their idolatry."

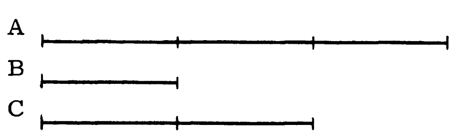
For Mersenne the entire matter of ranking the intervals according to their alleged superiority has to do with the extent to which the vibrations of the two sounds of each interval coincide, an approach perhaps first investigated by Giovanni Benedetti (1530-1590) and developed by Mersenne in correspondance with Descartes. *16 As Mersenne puts it:

> They are always more gentle when their vibrations or their movements are more often united, although many people do not derive much pleasure from them because of the preoccupation of the spirit, or the differences among imaginations, ears, and capacity which cause some to prefer a greater variety than others, a result due to different tastes. *17

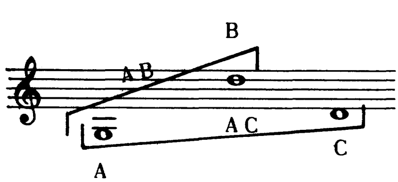
He elaborates upon this principle of coincidence by going on to state that the octave is half as consonant as the unison, the perfect fifth is one-third as consonant as the unison, and the perfect fourth is one-fourth as consonant as the unison. The more often the vibrations of the interval's two sounds coincide, the more "consonant" is the interval. Somewhat oversimplified, such a theory is nevertheless an important attempt at a physical explanation of consonance. *18

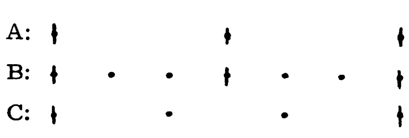
Mersenne explains this method of arriving at the rate of consonance in the following manner: he multiplies the terms of the ratio and then expresses the rate of coincidence in terms of the product. Thus the octave, whose frequency ratio is 2:1, has the vibrations of its two sounds coinciding at the rate of once in every two cycles of the upper note, a coincidence ratio of 1/2 (thus half as "consonant" as the unison). (This case, and those which immediately follow, will be explained more fully with graphic illustrations presently.) To continue, the perfect fifth, whose frequency ratio is 3:2, has the vibrations of its two sounds coinciding at the rate of twice in every six cycles of the upper note, or once in every three for a coincidence ratio of 1/3 (thus one-third as "consonant" as the unison). The perfect fourth, then, is one-fourth as "consonant" as the unison.

In the case of two or more intervals which have the same coincidence ratio in terms of the upper note, Mersenne favors the interval whose frequency ratio is the "simplest to comprehend". To illustrate this he gives the following diagram and commentary, comparing the perfect twelfth (3:1) with the perfect fifth (3:2), both of whose vibrations coincide once in every three cycles of the upper note (thus, both intervals are one-third as consonant as the unison):



The following transcription might clarify the above:





Mersenne explains:

> Let it be assumed that strings A and B begin to vibrate at the same time. While A makes one cycle, B will make exactly three, and when A begins its second cycle, B will begin its fourth. . . . But if A and C begin to vibrate at the same time, when A has completed its first cycle, C will make one-half of its second, and it will not be ready to begin again with A at the second moment [of A], but only at the third moment. *19

That is, strings A and B have their vibrations coinciding at every cycle of A, but strings A and C have their vibrations coinciding only at every two cycles of A. Mersenne therefore considers the interval of a perfect twelfth (AB) superior to that of a perfect fifth (AC), because the former has its vibrations coinciding more often in terms of the lower note (A). Mersenne adds:

> . . . if their sounds mix and unite more easily, their sweetness is greater, . . . and the greater the pleasure that one receives comes from this greater union. *20

Mersenne only appears to be inconsistent in his approach here. When Mersenne compares the octave, fifth, and fourth (see above), it suffices for him to compare their respective rates of coincidence in terms of the upper note of each interval; if, however, this process of comparison shows an equality of coincidence in terms of the upper note – as in the perfect fifth and perfect twelfth – he makes the comparison in terms of the lower note. Completing this procedure for all of the consonances under Mersenne's consideration produces the results shown in Table 1.

Elsewhere in the Harmonie universelle Mersenne gives yet another way of arranging the consonances in order of their "perfection" as consonances; he states that the intervals appear in their order of consonances if the terms of each interval ratio are multiplied – the smaller the product, the more consonant the interval. *21 The following is the result:

interval:	P8	P12	P5	M10	P4	M6	M3	m3	m6
ratio:	2:1	3:1	3:2	5:2	4:3	5:3	5:4	6:5	8:5
product:	2	3	6	10	12	15	20	30	40

TABLE

1

Interval	Ratio	Percentage of Coincidence for upper note	for lower note
P8	2:1	50%	100%
P12	3:1	33%	100%
P5	3:2	33%	50%
P4	4:3	25%	33%
M10	5:2	20%	50%
M6	5:3	20%	33%
M3	5:4	20%	25%
m3	6:5	17%	20%
m6	8:5	12 1/2%	20%

One will note that the order of consonances in the table above is identical with that found by the coincident method save in one respect: the major tenth appears "superior" to the perfect fourth in the table above, whereas the perfect fourth appears superior to the major tenth in the table of coincident percentage (Table 1), but only in terms of the upper note.

Despite the evidence found by either method, Mersenne feels that the perfect fourth is inferior to both the major and the minor third. He points out that, although the fourth appears earlier in the harmonic series than do the two thirds, the fourth is not allowed in two-part counterpoint and, in fact, is considered less desirable even as a dissonance there than is the seventh. That is, the 4-3 suspension is not as effective in two-part counterpoint as is the 7-6 suspension (as can be ascertained by an examination of sixteenth-century two-part music where the 4-3 suspension is rare *22).

Rationalizing the perfect fourth was, of course, a challenge accepted with widely varying results by most theorists before Rameau. For Mersenne the fourth is to be considered a consonance only because it is the "shadow" of the perfect fifth (to borrow Descartes' expression); in Mersenne's words the fourth is a "sterile" interval, "une fille bastarde", which produces nothing good either by division or multiplication. By this he means that the perfect fourth, divided arithmetically, does not produce two consonances: 4:3 (8:6) divided to give the arithmetic mean yields 8:7:6, a progression of ratios of no musical value. If the perfect fourth is multiplied by itself – that is, if a fourth is added to a fourth – the resulting interval is a minor seventh, a dissonance: 4:3 x 4:3 = 16:9. (In connection with this last process, however, it should be noted that no consonance except the octave produces another consonance when added to itself: e.g., two major thirds produce an augmented fifth.)

The major and minor thirds are discussed by Mersenne in words familiar to his day. These intervals are born of the division of the perfect fifth (6:5:4); the minor third is the "residue", and the major third the original interval. They thus have a relationship to each other like that between the perfect fourth and fifth. Also in favor of the major third is the fact that it can be produced by sympathetic vibration with relative ease – or more exactly, the compound major third is thus found (5:1).

The major and minor sixths are discussed in various ways: the major sixth is a residue after the minor third is taken from the octave, the minor sixth a residue of the major third; either quality of sixth produces a good effect when placed above or below its opposite-quality third; and either the major or the minor sixth can be used to resolve a dissonant seventh in counterpoint. Nevertheless, Mersenne remains uneasy about the consonance of these two sizes of sixth, because their interval ratios (5:3 and 8:5) are not superparticular as are the ratios of the other consonances.

Mersenne makes several interesting remarks during his consideration of the relative consonance of the thirds and sixths. His various methods of computing the coincidence or "consonance" of intervals result – as was seen earlier – in the same order for the following four intervals: the major sixth is most consonant, followed by the major third, the minor third, and the minor sixth. There is no doubt that the minor sixth is the least consonant; it is the residue after the major third is taken from the octave, but, for Mersenne as for Zarlino before him, the minor sixth is more closely associated with the minor third than it is with the major third. As Mersenne puts it:

> One compares this minor sixth with the minor third as to gentleness and agreeableness ordinarily, for they are of the same nature. *23

The computations of "consonance" bear him out: one can associate the minor sixth with the minor third (1/8 and 1/6 as consonant as the unison, respectively), although they are not as comparable as are the major sixth and major third (both of these being 1/5 as consonant as the unison).

Mersenne is consistent with his theories of consonance, if not quite orthodox, when he selects the major sixth as the original interval and the minor third as the residue in reference to the octave. The major sixth is found to be more consonant, and it occurs earlier in the harmonic series. Yet it continues to bother Mersenne that the ratio of the minor third (6:5) is "easier to comprehend" – being superparticular – than is that of the major sixth (5:3).

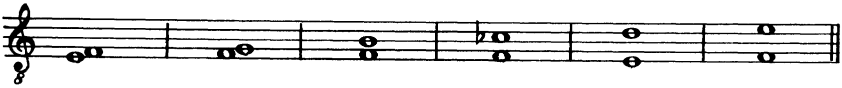
Although Mersenne did not arrive at a knowledge of the following aspects of consonance, there is some evidence that his preference for the major sixth can be defended. One should note that in each case shown in Example 2, the major sixth is "superior" to the major third: (a) in the proximity of the difference tone, (b) in the conjunction of upper partials, and (c) in the primacy of summation tones. One should also notice that "complete triads" are formed in the case of the major sixth only: (a) by the sixth and its difference tone, (b) by the sixth and its partials at the earliest conjunction, and (c) by the sixth and its first summation tone.

It might be of interest to point out in passing that if the seven prime consonances are considered in the light of their first summation tones, the more consonant intervals (following Mersenne) have summation tones which are "in tune" in the harmonic series and the less consonant do not, as shown in Example 3.

Mersenne turns once again to the harmonic series in order to determine the most effective register in which the various consonances can appear. Thus, he argues that the octave sounds best when placed low, the minor third best when positioned high, and so on. Also, he finds that the larger intervals which appear early (lower) in the series have a larger range of effective placement than do the smaller intervals which appear later (higher) in the series. This means that an interval like the perfect fifth may be placed with good effect within a wider tonal compass than, say, the minor third. This is borne out by the very presence in the series of no less than three perfect fifths (2-3, 4-6, 8-12 partials) against only two minor thirds (5-6, 10-12) within the first twelve notes of the series. Also operative in this connection is the fact that the lower an interval is in any given harmonic series the richer it will be in the quantity of upper partials — although this can be stated only in a most abstract way, without considering the innate partial-content of various timbres. (Indeed, throughout this study one must appreciate Mersenne's abstract, mechanistic view of musical acoustics.)

The Dissonances

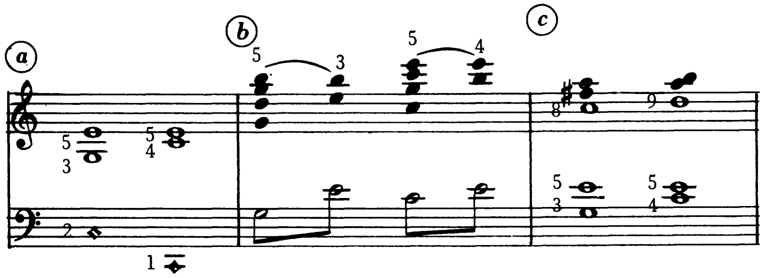
Mersenne adopts the orthodox number of dissonances for his day, and he gives their ratios as follows:



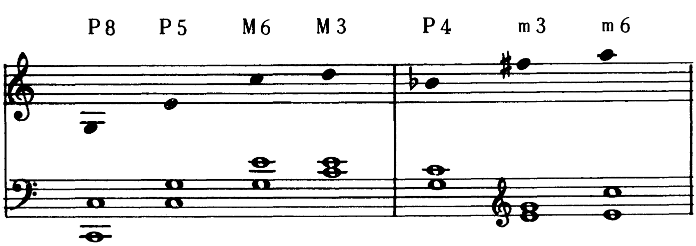
15:16 8:9 32:45 45:64 5:9 8:15

EXAMPLE

2



3



It is unclear why Mersenne omits the minor tone (9:10) from this list of dissonances, as well as the smaller minor seventh (9:16) which appears in other tables given by him; both of these intervals do appear above in their respective octave-inversions (5:9 and 8:9). The diminished fifth seems to be a combination of a Pythagorean minor third with a Just minor third: $\frac{32}{27} \times \frac{6}{5} = \frac{64}{45}$.

Of this "false" fifth Mersenne adds that it is "so little different from the tritone (32:45) that the ear can barely distinguish them" (being less than a syntonic comma larger). He remarks that in equal temperament the tritone and the false fifth are identical in sound; it is probably for this reason, to show the enharmonic equivalence of these two dissonances, that Mersenne gives the false fifth as f-c♭, rather than, say b-f.

Consonance and Dissonance Compared

Mersenne entertains some very interesting opinions regarding consonance and dissonance, especially in the light of his otherwise rather metaphysical bent. Considering what he has to say on this subject – in substance, that there can be no hard and fast line drawn between the category of consonance and that of dissonance – he appears to be something of a relativist.

Mersenne wonders, for example, if some superior ear (plus epuré) might admit as consonances such intervals as the seventh or ninth. He feels that it is difficult to justify the fact that the last consonance allowed is the minor sixth; why should the number 7 or the number 9 not be permitted to serve in the ratio of a consonance? Seven can be rationalized as the sum of 6 and 1, or 5 and 2, or 4 and 3 – all numbers used in the ratios of consonances; likewise, the number 9 can be rationalized. Furthermore, ratios such as 6:7 or 7:8 are as easily comprehended by the mind as is 5:6, each being a superparticular ratio.

To those who would say that the interval of a major or minor second hurts the ear or is otherwise disagreeable, Mersenne points out that this is but a relative judgment:

> The vibrations of the minor third and sixth begin to displease and very nearly approach dissonance, because they have more vibrations of the air which do not coincide at all than those which do, . . . and the displeasure that one experiences in hearing them is only a little less than that when hearing seconds, tritones, and so forth. *24

The accuracy of this statement can be shown by comparing the percentage of coincidence for these consonances with that for the less dissonant dissonances. The minor third is coincident in terms of its upper note 17%, and the minor sixth only 12 1/2%; whereas the minor seventh (5:9) is coincident 11%, as is also the major tone (8:9) – not a very large difference in percentages. (The various other dissonances are coincident, in terms of the upper note, as follows: minor tone, 10%; major seventh, 7%; major semitone, 6%; tritone, 2%; and false fifth, 1 1/2%.)

Mersenne contends, moreover, that if one grows accustomed to sevenths and ninths, these would eventually become "consonant". The same might even be true of such an interval as that of the defective minor third of the harmonic series (5:7). Elsewhere, he speaks of the tritone in the same way:

> But it is very difficult to judge if it is always disagreeable or not, due to the fact that if all the ears of the good masters are not in agreement, one can choose whichever side one wishes to follow, for even though there be one hundred who find this relation disagreeable, there may be two or three who judge it permissible and not disagreeable. Now it can be maintained that these last deserve to be followed in their opinion, for the plurality of voices does not rule in the physical realm as it does in the moral. *25

In the final analysis, Mersenne conjectures that all intervals might be arranged along a continuous spectrum, running from the "perfection" of the unison to the most disagreeable of the dissonances. Such a scale of values would be founded solely upon the degree to which each interval is coincident, be it consonance or dissonance:

> I say firstly that the dissonances which have as many vibrations of the air separately as the consonances have them together are as disagreeable as the aforesaid consonances are agreeable, since the gentleness or harshness of different sounds that are heard simultaneously must be attributed to the union or disunion of the vibrations of the air, which are the aforesaid sounds. *26

Despite the length and detail of his discussion, however, Mersenne reviews all of the various points that he has raised concerning consonance and dissonance, and he concludes with the comment: "Nevertheless, all of these reasons do not completely satisfy me."

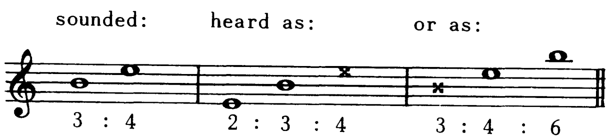
THE THEORY OF INVERTIBILITY

Mersenne's concept of the invertibility of intervals is closely associated with a process which he refers to as "representation". The following discussion will attempt to show just how Mersenne makes use of this process, and how it relates to the principle of octave-inversion.

In reading the Harmonie universelle one is convinced that Mersenne accepts the principle of octave-inversion as given by Zarlino before him. However, as Mersenne investigates this principle – especially in the light of the newly arrived-at harmonic series – he forms a deeper awareness of the implications. For one thing, his conception of the phenomenon is centered upon its aural aspects; in giving reasons why the ear mistakes certain intervals for others, he says:

> Another reason is founded upon the close resemblance between the two sounds of an octave, for it is so great that many err in judging these two sounds, mistaking the low notes for the higher, or the high note for the lower. Because of this, they judge the fifth as a fourth, and the fourth as a fifth. I have often noticed this, and I wish to explain it so that musicians might be on guard. *27

In giving his explanation of why the ear makes this unconscious octave-transposition, Mersenne develops his theory of "representation". In mistaking the perfect fifth for the perfect fourth, the ear hears the representation of the true interval by taking one of the notes either an octave below or an octave above its true position – due, as he says, to the close resemblance of notes an octave apart. Thus:



Expressed mathematically, representation occurs when one of the terms of an interval ratio is divided or multiplied by two.

Mersenne goes on to point out that, for this same reason of octave-resemblance, the minor sixth (5:8) is often mistaken for the major third $5:\frac{8}{2} = 5:4$). In fact, since the larger term of the minor sixth is unlikely to be heard without also hearing its representation an octave below, it follows that one cannot hear the minor sixth without at the same time hearing a major third below. Likewise, the sevenths have something of the same nature as the seconds, "and are ranked with them as those of which nothing good can be expected". This process of representation results, of course, in obtaining either the inversion or a compound form of the given interval.

Even more interesting is a further implication which Mersenne draws from the phenomenon of octave-inversion; he began to notice that there was some superiority among those consonances which have an even number as their lower term. Such intervals, when represented (i.e., their lower term divided by 2) appear in the compound form; whereas the other consonances, when so represented (i.e., their upper term divided by 2) appear in the form of inversions. In other words, the former consonances are not fundamentally altered by the process of representation, but the latter consonances are. As examples: the perfect fifth (2:3) is represented by the perfect twelfth (1:3) – lower term divided by 2; the perfect four (3:4), however, is represented by the perfect fifth, its inversion (2:3) – upper term divided by 2.

Mersenne also noticed that a consonance which has an even number as its lower term is more "agreeable" than one which does not, although there are exceptions as will be pointed out below. If the harmonic series is examined in the light of this principle of Mersenne, some support for his theory will be found. Any interval whose lower term is an even number (i.e., 4:5) will find representation of its lower note an octave below, within the harmonic series – thus creating the octave-compound (2:5). Conversely, any interval whose upper term is an even number (i.e., 3:4) will find representation of its upper note an octave below, within the harmonic series – but the interval thus created will be the octave-inversion (2:3). Therefore, such intervals as the perfect fifth (2:3) and major third (4:5) are more "agreeable" (more stable ?) and might be considered as "original" intervals; whereas such intervals as the perfect fourth (3:4) and minor sixth (5:8) are less agreeable (less stable ?) and might be considered as "inverted" intervals. *28

The major sixth (3:5) and the minor third (5:6) constitute a problematical pair, as to which is "original" and which is "inverted": neither of these related intervals has an even number as a lower term. Mersenne finds the major sixth more stable and points to its lower position in the harmonic series. (Perhaps he should have pointed out also that the minor third is more vulnerable to inversion inasmuch as its upper term is an even number, whereas the major sixth – having both terms as odd numbers – cannot be inverted or compounded by representation.) Mersenne's selection of the major sixth over the minor third does not agree, of course, with what many believe; that there is at least some evidence to support his view, however, can be shown by referring to Example 2 and the accompanying discussion.

It should be noted in connection with Mersenne's theory of representation that the interval in question must have its ratio reduced to lowest terms. Moreover, the octave, which is certainly stable, does not have an even number as its lower term and is therefore an exception of sorts. (More inclusively, perhaps, is the view that the "original" intervals have lower terms which are successive powers of 2: the octave, 2^0; the fifth, 2^1; the major third, 2^2; although this still does not explain the major sixth.)

Although Mersenne, in writing about representation, seems to be thinking chiefly of the consonances, there is some evidence that he was aware of the application of this principle in grading the dissonances. He points out, for example, that the major tone (8:9), the tritone (32:45), and the major seventh (8:15) are the three principal dissonances corresponding to the three principle consonances (the octave, fifth, and major third). These three dissonances, significantly, have even numbers as their lower terms and are thus more "agreeable" than the other dissonances.

CONCEPTS OF HARMONY

Mersenne reflects the early Baroque aesthetic when he makes it quite clear that he values monophonic music over polyphonic music. He writes:

> One has much difficulty convincing composers that the simple rendition of songs is more agreeable than when they are sung in two or more voices, because they fear their compositions would be discredited, as they would be, in fact. *29

This negative view of polyphony is maintained as he goes on to assert that polyphony was introduced "100 or 200 years ago" to compensate for the failure of Western music to maintain the melodic genius of the ancient Greeks. He then sets forth numerous "proofs" for the superiority of the simple monophonic song, which can be summarized thus:

(a) it is easier to appreciate a single voice;
(b) it is easier to understand the text;
(c) they do not have the disunity and conflict which is found in polyphonic music;
(d) the simple is superior to the complex, just as bread and water are superior food and drink.

In a characteristic gesture of impartiality, however, Mersenne also sets forth various "proofs" for the value of music in more than one part. Summarized, these are:

(a) a single vocal line seems bare;
(b) variety is always interesting, just as a bouquet of several varieties is superior to a single flower;
(c) nature shows us that everything is constituted of different and various parts;
(d) it is unreasonable to assume that so many in the past could be wrong in judging polyphony to be superior.

There is one point which he raises that is of particular interest here: he suggests that consonances like the fifth and fourth sound better melodically than harmonically, that this is even more true of the dissonances, and that therefore the melodic holds some sway over the harmonic. Concerning this entire question, however, Mersenne concludes that at best the arguments remain problematic.

When Mersenne does turn his attention to music in more than one part, he follows in the tradition set by Zarlino. He favors three-part texture to two-part for the mathematical reason that three-part writing calls upon proportions of three terms, whereas two-part writing employs only simple ratios. Furthermore, three-part texture always contains two ratios of different kind and their proportion is formed into what he calls the harmonie parfaite, the major or minor triad. As for four-part texture, Mersenne admits that such writing is even more agreeable — even though the octave is a duplication of one of the tones — for it gives a greater harmony and "fills the ear".

Mersenne, in his customary thoroughness, presents a lengthy discussion of the relative superiority of the bass part over the other parts. Yet, there is a subtle shift away from the Renaissance emphasis upon the bass (as in Zarlino) and a turning toward the Baroque concern for the beauty of the soprano line. This rather new conception is shown when he writes concerning the uppermost part:

> . . . it is the ornament and the beauty of the ensemble, that it pleases even more when it is sung alone than if the other three parts are also heard. . . . Moreover, it seems that the bass and the other parts were invented only to accompany and enrich the soprano as the principal subject of music. *30

Nevertheless, he is convinced of the importance of the bass in governing the conjunction of the musical lines. Like Zarlino and Descartes, he attributes this primacy to the gravity and simplicity of the bass, and to the length and fewer vibrations of the lowest strings.

It is significant that Mersenne detects the relationship of the lowest tone to those above it; he points out:

> The bass string contains the upper string, just as unity contains the binary, quaternary, etc. . . . The low sound can be considered as the whole, the upper sound as a part inasmuch as it is formed by the division of the lower, . . . and the other parts are like fractional numbers; but the whole number is the foundation of the fractional numbers. *31

Even more significant is the manner in which Mersenne relates these harmonies to the harmonic series:

> And if one considers the order of the sounds of the trumpet, it is a simple matter to conclude that the bass is the foundation of music, since the foundation of all the sounds of this trumpet is the lowest, after which it climbs to the octave, and from the octave to the fifth, etc.

He goes on to point out that, although the uppermost part is significant as the musical subject of one's attention, it is the bass which has the greatest effect at cadences and always serves as the "fondement de l'harmonic".

Reflecting the new Baroque emphasis upon the polarity of the outer vocal lines, Mersenne deviates from Zarlino's opinion on the order in which the parts of a polyphonic composition should be written. Rather than beginning with the tenor, Mersenne feels that:

> It seems better to me to begin the composition with the bass and with the top voice at the same time (although good teachers condemn this manner of composing), since they form the concords whose terms are the farthest removed, so that one then has nothing else to do in order to add the other parts except to divide these concords. *32

Clearly, the above passage is an unequivocal support of a two-voice framework — a melodic line and a bass line, with inner parts merely filling in between — an approach which came to characterize the so-called age of the figured bass. *33

One cannot proceed very far in developing a full and certain idea of Mersenne's conception of harmony, chiefly because Mersenne does not go into any great detail. For these harmonies he refers the reader to the table given by Zarlino (Istitutioni harmoniche, p. 241).

SCALES AND MODES

Mersenne continues the Renaissance tradition of discussing, in some detail, the various tetrachord patterns of the Greeks, the medieval hexachord systems, the Guidonian hand, and the three genera. For the most part, there is little that is new in these discussions, except for his attempts to broaden the sol-fa system to cover the seven-note scale (suggesting ci for the whole step beyond la).

In connection with the gamut, however, there appears one of Mersenne's most important pioneering ventures: the establishment of a given frequency for each pitch. His work in this area has rightfully received adequate study elsewhere and need not detain us here. *34

As for the twelve modes there are numerous small remarks that tend to suggest something of an impatience with the subject or a desire to cover it merely as a fading, theoretical system. Mersenne prefers to lean upon the work of others, recommending to the student such other treatises as take up the subject thoroughly.

In this connection, Mersenne makes use of a term which had not appeared in this sense within music treatises when he explains that, in order to identify the modes, it suffices to indicate the position of the "root" (racine)*35, but there is no elaboration of the possible implications of this term. Elsewhere, concerning the order of the modes (he follows the revised ordering of Zarlino, with the first mode on C) Mersenne remarks very justly that it does not actually matter where the modes are begun; he emphasizes that it is the semitonal pattern of each mode that is characteristic, thus stressing the transposition of modes as a reality — a point often overlooked in treatises of the day. He concludes his treatment of the modes with short polyphonic examples of each mode, written in two voices and conservative in nature.

More importantly, Mersenne discusses several ways in which the twelve modes might be reduced in number. Almost as if he feared criticism for his considering such a bold step, he assures the reader:

> One must not think that I wish to do away with the twelve modes, or to reproach those who have established them. *36

Nevertheless, he explores the possibilities.

At one point Mersenne remarks that it is often difficult to distinguish modes II, VII, VIII, IX, and X from mode I (i. e., the plagal mode on C and the authentic and plagal modes on F and G), inasmuch as each features a major third above the final. In one respect, of course, he is simply reiterating a similar point made by Zarlino concerning the major-minor quality of the modes, but he seems to be going a step farther by overlooking other distinguishing features of these modes (i. e., cadences, semitonal patterns). Either he is being insensitive to the various modal characteristics, or he is considering them to be inconsequential or customarily eliminated through ficta. Elsewhere he observes that the more similar the semitonal pattern of the modes are, the more alike they are in their effect; thus, mode I is more comparable to IX (on G) than it is to III (on D). He also points out, unlike his predecessors, that if one considers all combinations of the two sizes of major second (in just tuning) there would actually be seventy-two modes.

As Mersenne considers the seven octave species (i. e., the seven patterns of tones and semitones found between each "white key" and its octave), he decides that:

> These twelve modes have not been established for very good reasons, inasmuch as the same species of fifth and

> fourth which form the first seven modes are found again in the same order in the last five; . . . it is unnecessary, therefore, to augment their number. . . for fear of an imaginary distinction which depends only on the name of the clefs and letters of the scale. *37

That is, the authentic-plagal distinction is meaningless. The authentic modes on C, D, E, and F (I, III, V, and VII) together with the plagal forms of the first three (II, IV, and VI which begin on G, A, and B) exemplify all seven of the possible semitonal patterns to be found in an octave. He does not press the point, however, but gives passing recognition to the differing cadence points for the various modes. (It will be La Voye-Mignot who will write clearly against the plagal distinction, in 1656.) *38

Further, in a passage heretofore given no special attention, Mersenne suggests that there might be only two distinct modes. He writes:

> Thus it turns out that certain modes have more resemblance with some than with others. It suffices to interchange the seven species of octave for the seven tones or seven principal modes whose four cadences or modal tones reduce to ut, mi, sol, fa or. . . to re, fa, re, sol, . . . For, although one forms cadences on mi, sol, mi, la in the third species of octave, these have no other force nor intervals than the re, fa, re, sol of the second [octave species] or the re, fa, mi, la of the sixth species – just as the fa, re, fa, fa of the fourth is nothing other than the ut, mi, sol, fa of the first species.
>
> From whence one may conclude that there are only two modes which are different in their cadences or principal notes. . . . *39

Mersenne is here comparing the cadence points of the "seven principal modes" (i. e., those corresponding to the seven octave species) in the sol-fa terminology of the day. Example 4 provides an illustration which might assist the reader in deciphering Mersenne's important remarks.

It is surprising that Mersenne's remarks, which come so close to establishing the major-minor system, have been given so little attention by modern commentators. The following additional comments will attempt to indicate to what extent Mersenne foresaw the two-mode system, although the pertinent section of his Harmonie universelle is one of the most difficult to comprehend.

Mersenne, in continuing his discussion of the two essentially different modes, relates the matter to a proposed revision of the Guidonian hexachord system. He realizes that the so-called hexachord "of nature" (c to a), if extended to the octave, contains the same interval pattern as the so-called "soft hexachord" or B♭ hexachord (f to d, with b♭), if likewise extended to the octave. This being so, he then recommends a revision of the sol-fa system which would include additional syllables to complete the octave and, by using different added syllables, would reveal the difference between the two extended hexachords which are distinguishable. He would also drop the hexachord of nature since it becomes, thus, superfluous. See Example 5a. Although all three scales have a semitone between the third and fourth degrees, the B♮ hexachord differs from the other two by having its second semitone between the sixth and seventh degrees instead of between the seventh and eighth.

Mersenne then revises the sol-fa system, based upon the two forms of extended hexachords which are distinguishable — namely, the B♭ and the B♮ hexachords. He recommends the addition of the syllable ci for the whole step beyond la in the B♭ hexachord and bi for the half step beyond la in the B♮ hexachord as shown in Example 5b.

Then, Mersenne makes a most interesting decision; he relates these two syllable patterns to the twelve modes. He says:

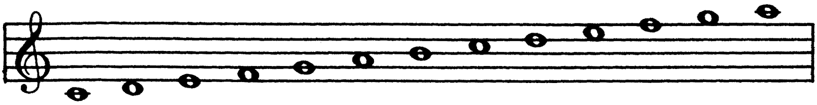
> Therefore, it is easy [!] to see that the twelve modes reduce themselves to the signs or characters of b♭ and of b♮, as can be shown more fully by the sol-fa patterns of each mode, by setting forth the ambitus of the notes or pitches, with the letters necessary to understand them. *40

Unfortunately, Mersenne does not show his procedure at all clearly. What follows, below, must of necessity be an approximation of his ideas on this point.

Following some calculations attributed to Kepler, he sets forth two gapped scales, each supplied with string ratios (which are only representative of the vibration ratios):

EXAMPLE

4

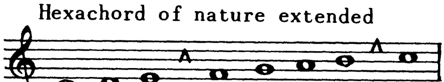


1	ut		mi		sol			fa					
2		re		fa		re			sol				
3			mi		sol		mi			la			
4				fa		re		fa			fa		
5					ut		mi		re			sol	
6						re		fa		mi			la
7	(defective)												

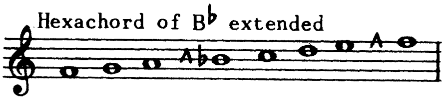
5

a

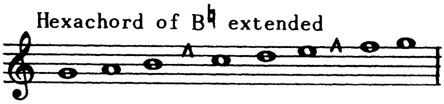
Hexachord of nature extended



Hexachord of B$^\flat$ extended

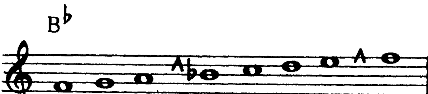


Hexachord of B$^\natural$ extended



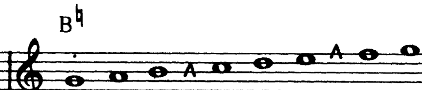
b

B$^\flat$



ut re mi fa sol la ci ut

B$^\natural$



ut re mi fa sol la bi ut

D	ut	12		G	ut	30
B♭	bi	15		E	la	36
A	la	16		D	sol	40
G	sol	18		C	fa	45
F	fa	20		B	mi	48
D	re	24		G	ut	60

If these two gapped scales are written out in the following manner they can be seen as models of the minor and major modes (See Example 6). By omitting the second and seventh scale degrees, Mersenne ignores the distinctive characteristics of the Phrygian, Mixolydian, and Aeolian modes (using today's nomenclature), and if the lowered fourth degree in Lydian is assumed – as it often was by Mersenne's time – there remain only two scales that are distinguishable from each other: one characterized by a re-fa (minor) third, the other by an ut-mi (major) third. (This is perhaps the earliest appearance of the Dorian with B-flat rather than Aeolian as prototype of the minor scale.) A half-century elapses before the major-minor system is recognized with greater clarity; Etienne Loulié writes in 1696:

> When a piece of music ends on ut, the mode is called major. When the piece of music ends on re, the mode is called minor. *41

CONCLUSION

It may be helpful, in closing the present study, to review the major points of Mersenne's theories which relate to the evolving tonal consciousness of theorists before Rameau.

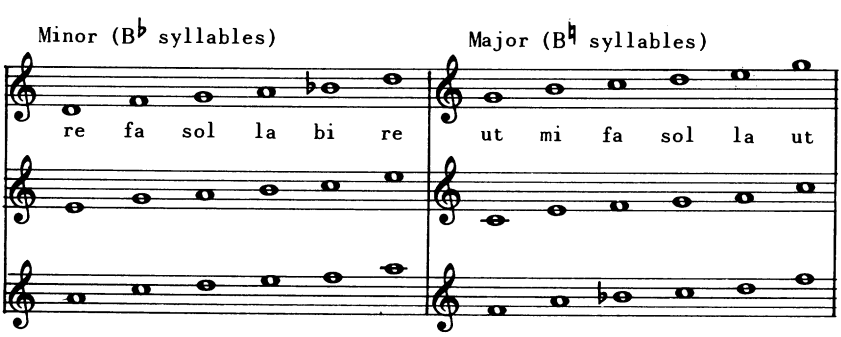
Fundamental in Mersenne's considerations is his influential emphasis upon vibration rates in discussing pitches and intervals. Of special significance is his emphasis upon the harmonic series as a basis for harmonic generation and for determining the relative consonance of intervals. Similarly important is the manner in which Mersenne related the consonance of intervals to the rate of coincidence of the two vibration forms, thus joining the vanguard in replacing the ancient metaphysical-numerical approach with one founded upon acoustical principles. He also advocated a continuum of relative consonance which tend-ed to do away with the rigid division of intervals into perfect and imperfect consonances and dissonances inherited from the past.

Mersenne's theory of interval "representation" also emphasized and explained the close acoustical relationship of each interval and its octave-inversion, an important step toward Rameau's recognition of chordal inversion. Reflecting the new aesthetic, Mersenne also stressed the importance of the outer voice lines in music of more than one part, founded on the acoustical properties of the harmonic series.

Although conservative in many respects, Mersenne was progressive in initiating attempts to modernize the ancient hexachord and sol-fa systems, and in suggesting the reduction of the twelve modes by eliminating the plagal distinction. Finally, through these efforts, Mersenne realized, however imperfectly, the fundamentals of the two-mode, major-minor system which has governed Western music to the present century. In short, Mersenne played a significant role in setting the stage for Rameau's remarkable achievements in the succeeding century.

EXAMPLE

6



R E F E R E N C E S

1 Marin Mersenne. Harmonie universelle contenant la théorie et la pratique de la musique (Paris, 1636), facs. ed. (Paris: Centre National de la Recherche Scientifique, 1965).

2 This treatise has been translated as Harmonie universelle: The Book on Instruments, transl. by Roger E. Chapman (Hague: Nijhoff, 1957).

3 For background to this matter see Claude Palisca, "Scientific Empiricism in Musical Thought", in Seventeenth Century Science and the Arts, ed. by Hedley Howell Rhys (Princeton University Press, 1961).

4 But see Willi Apel, "Arithmetic and harmonic mean", Harvard Dictionary of Music, 2nd ed. (Cambridge: Harvard University Press, 1969), p. 53. Apel's reference to Zarlino here is misleading; Zarlino regarded the harmonic mean of the octave as a fifth, the arithmetic mean as a fourth (Istitutioni harmoniche, 1558 ed., p. 309).

5 Palisca, op. cit., pp. 97-98.

6 "Treatise on Instruments", p. 250.

7 Ibid., p. 251.

8 Ibid., p. 208.

9 Ibid.

10 Ibid., p. 211.

11 Ibid., p. 210.

12 Ibid., p. 211.

13 Zarlino. Istitutioni harmoniche, 1558 ed., p. 155.

14 Descartes. Musicae compendium (Utrecht, 1650), p. 21. See also Descartes' letter to Mersenne quoted in Palisca, op. cit., pp. 111-112.

15 "Treatise on Instruments", p. 20.

16 Palisca, op. cit., pp. 104-110.

17 "Treatise on Consonances", p. 105.

18 The rate of coincidence must not be confused with the phonomenon called "beating".

19 "Treatise on Consonances", p. 65.

20 Ibid.

21 Ibid., p. 80.

22 Soderlund. Direct Approach to Counterpoint in 16th Century Style (N.Y.: Appleton-Century-Crofts, Inc., 1947), p. 27.

23 "Treatise on Consonances", p. 80.

24 Ibid., p. 87.

25 Ibid., pp. 313-314, a remarkable view of the moral code for a seventeenth-century Franciscan.

26 Ibid., pp. 129-130.

27 Ibid., p. 70. Compare with Zarlino's explanation in the Dimostrationi harmoniche (1571 ed.), pp. 89-90.

28 This comes close to being a basis for determining interval "roots", although Hindemith is probably correct in remarking that he found no treatise which states the idea of an interval having a root (Craft of Musical Composition, I, p. 68).

29 "Treatise on Consonances", p. 197.

30 Ibid., p. 207.

31 Ibid., p. 208.

32 Ibid., p. 277.

33 The figured bass came relatively late to the French theorists – not until the late seventeenth century.

34 See Hames Murray Barbour. Tuning and Temperament: a Historical Survey (East Lansing: Michigan State College Press, 1951).

35 "Treatise on Consonances", p. 291.

36 Ibid., p. 194.

37 Ibid., p. 183.

38 Traité de musique, pour bien et facilement apprendre à chanter et composer (Paris: Robert Ballard, 1656), pp. 77-78.

39 "Treatise on Consonances", p. 187.

40 Ibid., p. 190.

41 Elements or Principles of Music, transl. and ed. by Albert Cohen (Brooklyn: Institute of Mediaeval Music, 1965), p. 69.